COMMENTS ON THE PHYSICS OF
A NOVEL DIAMOND-SHAPED HYDRAULIC ACTUATOR

1) Introduction

Mr David Strain has developed and patented a hydraulic actuator having a geometry described as *diamond-shaped* and which, he claims, is "more efficient" than a conventional actuator: one using a cylindrical piston. In using the term *more efficient,* he asserts that this actuator "achieves more work than a conventional [cylindrical] piston with equal fluid pressure, at a lesser fluid volume" (Ref. 1, p. 10) I understand him to also claim that, when suitably configured with other hydraulic components, this device can be made to work as a motor without the input of energy from an external source (Ref. 2). He has built two sets of apparatus to demonstrate his ideas; the first raises a known weight and the second is configured as a motor. I have seen the first in operation, but not the second. His remarkable claims, if justified, questions the validity of physical principles having a long history of successful prediction. This note describes what I believe to be the relevant current physics, and applies it to the first apparatus in order to compare predictions with data he has obtained from that apparatus. Following this I recommend a possible course of action.

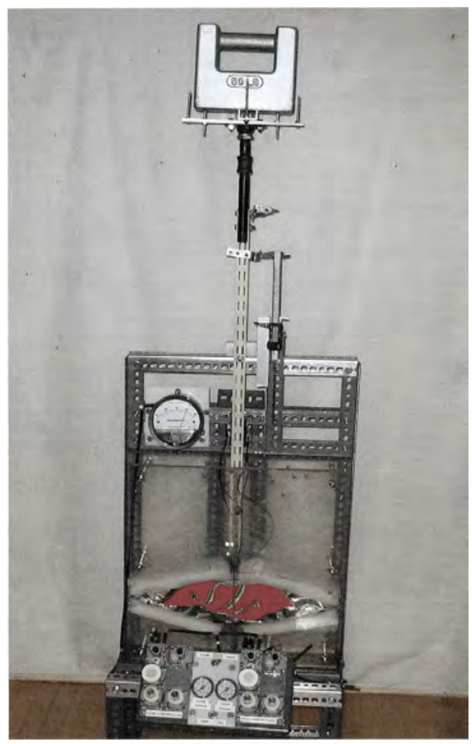

Fig. 1 Strain's weight-raising apparatus, showing diamond-shaped piston

Figure 1 is a copy of his photograph of the first apparatus; it uses the actuator to lift a weight. It comprises four rectangular plates connected by hinges at their ends, with the bottom hinge being attached to the apparatus base and the top to the load weight. The pressure driving the actuator is contained in a flexible plastic bellows further constrained by clear plastic side-plates. Strain claims that the work required to raise the weight through a specified distance is less than that required by a conventional cylindrical piston actuator (Ref. 3).

The analysis I present here, based as it is on conventional physics, predicts a pressure required to raise the weight significantly greater than that reported by Strain (Ref. 3), thus ostensibly supporting his claim. However the revolutionary implication of this claim is almost certain to be greeted with great skepticism by those familiar with the relevant physics. Consequently, in a previous communication I suggested that his next step forward was to have the entire experiment duplicated by others, with this duplication involving construction of new apparatus. But I believe it is extremely unlikely any Canadian scientific group—in universities or in government—would volunteer to take on the necessary work as part of an existing research project or government-sponsored industrial development programme, so that the work would have to be privately funded. Assuming that this is performed on a consulting basis by qualified research and engineering personnel, I estimate the cost could be about $50,000 or greater. Hence although certain additional tests might be performed on the apparatus of Fig. 1, I believe Mr Strain's only realistic choice is to demonstrate the operation of his motor while performing a measurable amount of work.

2) Comments on the relevant physics

Although Ref. 2 refers to the "Laws of Thermodynamics" in describing Strain's anomalous results, no heat processes appear to be involved. Since these laws govern the relationships involved in generating heat by work and work from heat, the relevant physics is not thermodynamics, it is the hydrostatics of incompressible fluids. The fundamental law here is Pascal's Principle: *the pressure at any point in a fluid acts equally in all directions*. The discovery of this law was a consequence of what was known in the early development of this subject as the *hydrostatic paradox*. The origin of this idea is illustrated in the adjacent Fig. 2, which depicts a flat-based narrow-necked vessel suspended on a weigh-scale and filled with a liquid of density *ñ* to a height H. With g being the acceleration due to gravity, if the atmospheric pressure is P_a and A is the area of the vessel's base, then the liquid exerts a downwards force of magnitude (*ñ*gH + P_a)A on the base. With the surrounding atmosphere exerting an upward force of magnitude $P_a A$, the resultant force on the base is greater than the weight of the liquid in the vessel. The paradox is resolved if the pressure exerted by the liquid on the sides of the vessel is everywhere normal to the sides. It can be shown that this generates an upwards tension in the vessel's sides which, at the connection to the base, exactly counterbalances the excess force generated by the liquid on the base.

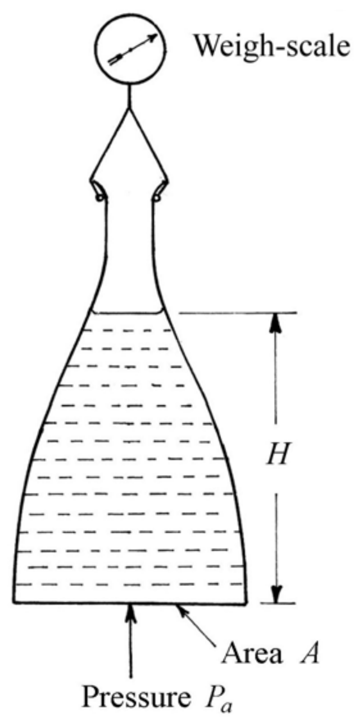

Fig.2

To apply this principle to hydraulic actuators, consider a piston and cylinder connected to a vessel which can change its shape and volume under the action of a fluid pressure p as depicted in Fig. 3. If the piston-vessel assembly is filled with a liquid which is virtually incompressible the vessel's increase in volume must equal the volume pumped in by the cylinder. That is, $V = Ax$.

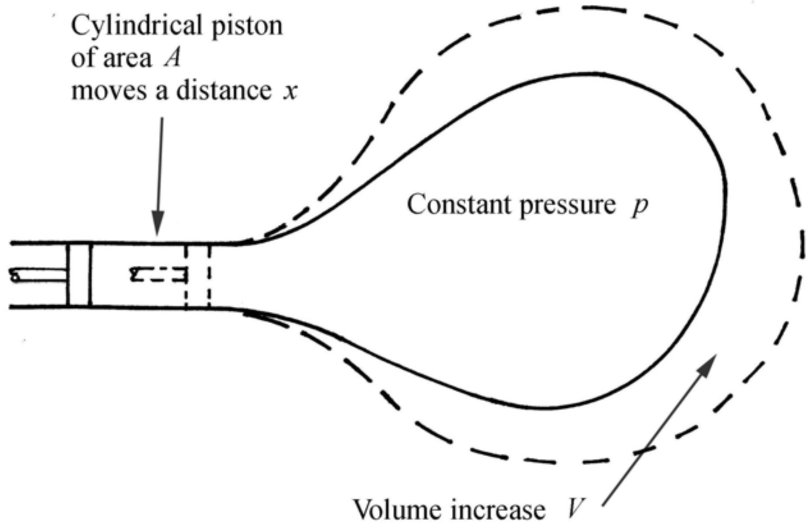

Fig. 3

If, in addition the piston pumps the fluid into the volume sufficiently slowly, the pressure p can remain constant throughout the entire assembly. Hence, while p may vary in time during pumping, the pressure acting on the vessel's wall will at every instant equal that acting at the piston face. If p also remains constant in time, the work W done by the piston is $W = PAx = PV$. If the process is sufficiently slow then fluid frictional forces acting both internally and at the vessel's wall are negligible and the work done by the piston must equal the work done by the pressure p in extending the vessel's walls. This is an application of the *principle of conservation of mechanical energy*.

This result can also be established by the following argument. Consider the work done by pressure on a plane surface of area A as it moves a distance h normal to itself while also moving laterally a distance d as depicted in Fig. 4. By Pascal's principle the pressure force $F = pA$ acts normally to the surface and, since the latter moves by h, the work done by p is $W = pAh$. But, from geometry; the volume V of the oblique cylinder swept out by the movement of the surface is independent of d and equal to $V = Ah$. Thus $W = pV$ for this volume. By breaking the volume increase in Fig. 3 into a large number of elementary volumes and adding the result gives, in the limit of sufficiently small elementary volumes, $W = pV$ again[2].

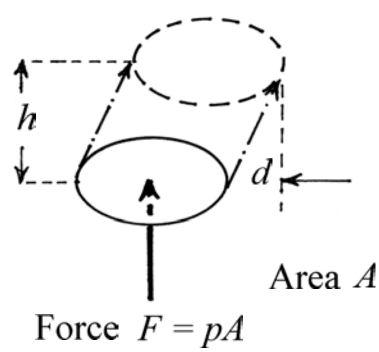

Fig. 4

3 Application to diamond-shaped actuator

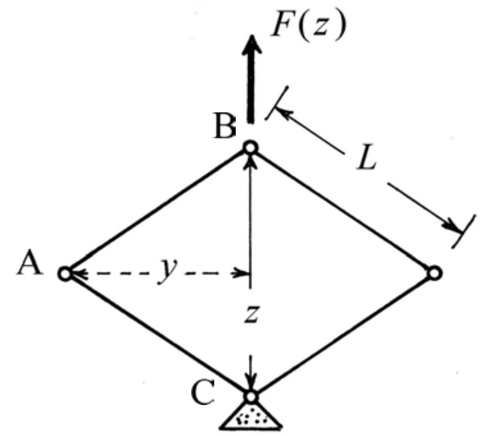

Fig. 5

With the relevant geometry depicted in Figure 5, each of the four plates has length L and width B. Since the triangle ABC has area $\frac{1}{2}zy = \frac{1}{2}z\sqrt{[L^2 - (z/2)^2]}$ the volume of fluid V_F in the actuator is

$$V_F(z) = \frac{1}{2}Bz\sqrt{4L^2 - z^2} \qquad (1)$$

Omitting for the moment the effect of gravity, if the actuator height increases from z to $z + h$ with the p held constant, then the work W_F done on the actuators wall is $p[V_F(z + h) - V_F(z)]$. In the absence of extraneous forces such at that caused by friction in the mechanism, this equals the work W_E produced by the force $F(z)$ generated by the actuator. For sufficiently small h, this external work $W_E = F(z)h$ approximately, with the approximation becoming increasingly accurate as h tends to zero. Hence

$$F(z) \rightarrow p\left\{\frac{[V_F(z + h) - V_F(z)]}{h}\right\} \quad \text{as } h \rightarrow 0$$

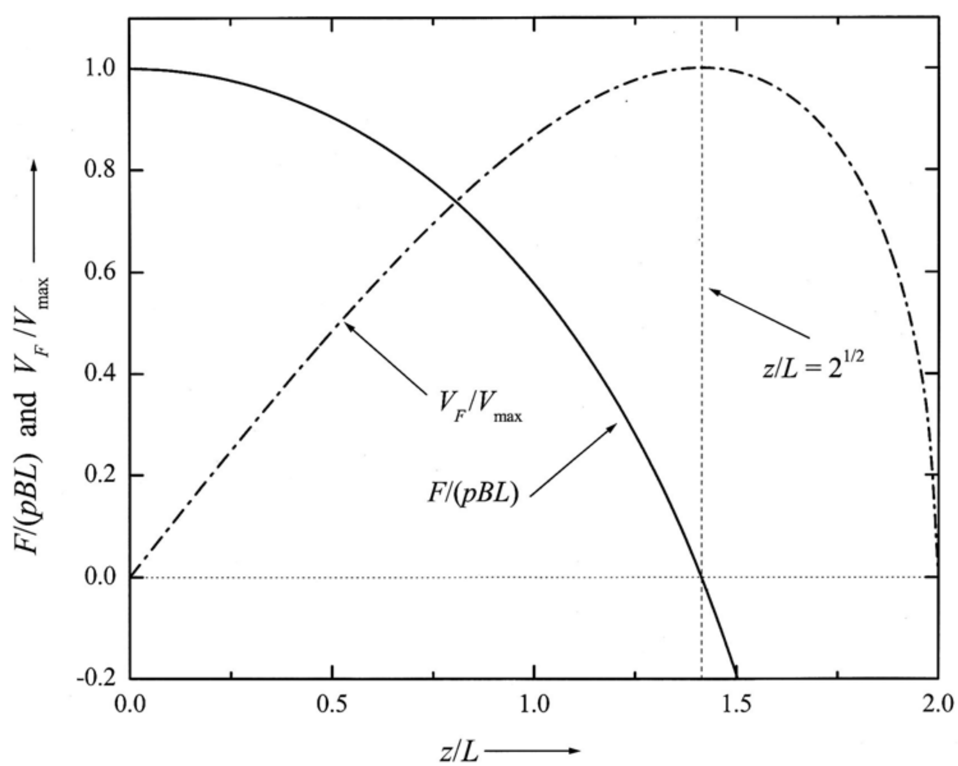

Fig. 6

But the quantity in the {} brackets in the above expression approaches the derivative $\mathrm{d}V_F/\mathrm{d}z$ as h approaches zero. Carrying through the differentiation of equation 1 gives

$$F = p\frac{\mathrm{d}V_F}{\mathrm{d}z} = p(z)BL\left\{\frac{2 - (z/L)^2}{\sqrt{4 - (z/L)^2}}\right\} \tag{2}$$

which gives $F = pBL$ at $z = 0$. Note also that, while p is held constant as the actuator height increases by a small amount h, this result also applies when p varies with z.

Figure 6 is a plot of equations (1) and (2) in normalized form; that is z is expressed as a fraction of L, and F and V_F are expressed as fractions of their maximum values, namely pBL and BL^2 respectively. Note also that V_F is a maximum and $F = 0$ when $z = \sqrt{2}L$; this is as expected, since the actuator plates form a square at this z.

3) Conventional analysis of weight-raising apparatus.

The following analysis is based on the principle of conservation of mechanical energy, which assumes that the work done in pumping an incompressible fluid into the actuator equals the work done in raising:

(1), the test weight with its supports;

(2), the mechanical parts of the actuator; and

(3), the work done in elevating the fluid in the actuator.

The latter two contributions are expected to be small, but they are easily included and provide a step in the direction of resolving discrepancies between the experimental results and the present theory. The mass of the test weight together with its supporting structure is M_W and its centre of mass is located at $z = H_W$.

The mass of the actuator is M_A and its centre of mass is assumed to be located at the centroid of its geometry, which is at $z/2$. With the volume of fluid in the actuator given by equation (1) and with its centroid also located at $z/2$, the potential energy of the system is

$$E_P(z) = gM_W(z + H_W) + gM_A\frac{z}{2} + \rho_F g V_F(z)\frac{z}{2}, \tag{3}$$

where $\tilde{n}_F$ is the density of the actuator fluid.

According to the principle of conservation of mechanical energy, and on applying the differential calculus, if the load is displaced upward by a sufficiently small amount h, the external work W_E that must be supplied is given by

$$W_E = E_P(z + h) - E_P(z) \approx g(M_W + \tfrac{1}{2}M_A)h + \tfrac{1}{2}\rho_F g\left\{z\frac{\mathrm{d}V_F}{\mathrm{d}z} + V_F\right\}h$$

If the external work W_E is supplied by a conventional piston-cylinder having the same elevation as the base of the actuator, namely $z = 0$, then $W_E = p_C V_C$. But if all the fluid in the cylinder is pumped into the actuator, then V_C equals the increase in V_F in the actuator, which for small enough h is $h(\mathrm{d}V_F/\mathrm{dz})$. Hence

$$g(M_W + \tfrac{1}{2}M_A)h + \tfrac{1}{2}\rho_F g\left\{z\frac{\mathrm{d}V_F}{\mathrm{d}z} + V_F\right\}h \approx p_C\frac{\mathrm{d}V_F}{\mathrm{d}z}h,$$

with this expression becoming increasingly accurate as h tends to zero. Cancelling the common factor h and rearranging gives

$$p_C = \frac{g}{\mathrm{d}V_F/\mathrm{d}z}\left[M_W + \tfrac{1}{2}M_A + \tfrac{1}{2}\rho_f\left\{z\frac{\mathrm{d}V_F}{\mathrm{D}z} + V_F\right\}\right] \quad (4)$$

Introducing V_F and $\mathrm{d}V_F/\mathrm{dz}$ from equations (1) and (2) gives the result we require. After suitable algebra, and with $y = z/L$, this is

$$p_C = \; = \frac{g}{BL}\left[M_W + \tfrac{1}{2}M_A\right]\frac{\sqrt{4 - y^2}}{2 - y^2} + \tfrac{1}{4}\rho_F g L\,\frac{y[8 - 3y^2]}{2 - y^2} \quad (5)$$

To compare Strain's measurements with the prediction of equation 5, I used the data given in Ref. 3, which gives M_W = 59.75 lbs = 27.11 kg, L = 9.125 in and B = 2.625 ins. To obtain a first estimate of M_A, I estimated the thickness of the plates from Fig. 1, and assumed that they are made from nylon at a relative density of 1.3, obtaining $M_A \approx 3.00$ kg. I assumed that the actuator fluid was a silicone oil with a relative density of 0.76. Figure 7 compares the predictions of equation 5 with Strains data from Ref. 3.

4) Discussion and conclusion

Figure 7 compares the estimates of equation 5 with the data of Ref. 2. As compared with the calculated results including the effects of both actuator and fluid weight, the measured pressures are significantly lower, being about 26% low at z/L = 0.25, decreasing to 19% low at z/L = 0.42. Since the possible effects of such sources of discrepancy as solid and fluid friction might be expected to increase the p_C required to lift the weight—at least in the process of raising the it—

this appears to support Mr Strain's claim.

[Further discussion of possible sources of experimental discrepancy leading to the conclusion that demonstration of the device as a motor performing useful work is the best way forward]

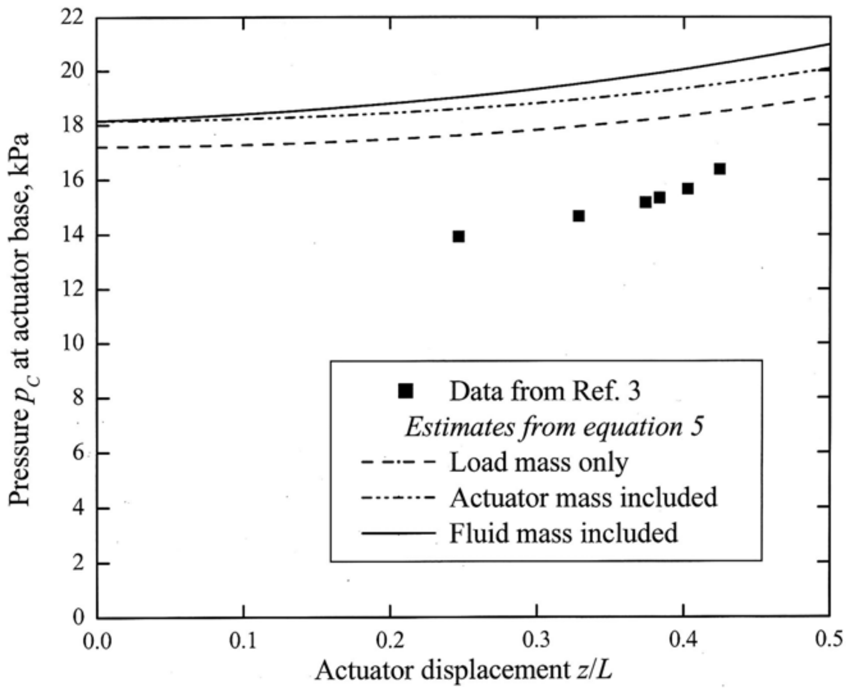

Fig. 7

5) References

1) Strain, D. (undated) "Scientific/technical observations and opinions via several professionals regarding the diamond-shaped fluid powered linkage, system and engine." Analysts of Pneumatic Systems Limited report.

2) Strain, D. (Undated) "A new view of thermodynamics relating to the working capability of pressurized fluids." Analysts of Pneumatic Systems Limited report.

3) Strain, D. (Undated) "Assessment regarding the fluid equation W= PV (work = pressure×volume) as it relates to the working capability of pressurized fluids applied in the conventional piston actuator and the diamond-shaped actuator." Analysts of Pneumatic Systems Limited report.